Physics 402
Fall 2022
Prof. Anlage
Discussion Worksheet for 31 August, 2022

1. The electron in a hydrogen atom occupies the combined spin and position state

$$\Psi = R_{21}(r)\left(\sqrt{\frac{1}{3}}\, Y_1^0(\theta,\phi)\ \chi_+ + \sqrt{\frac{2}{3}}\, Y_1^1(\theta,\phi)\ \chi_-\right)$$

a) If you measured the orbital angular momentum squared (L^2), what values might you get, and what is the probability of each?
b) Same for the *z* component of orbital angular momentum (L_z)
c) Same for the spin angular momentum squared (S^2)
d) Same for the *z* component of spin angular momentum (S_z)
e) What is the energy of the Hydrogen atom in this state?

28. Eigenfunctions for a rigid dumbbell rotating about its center have a ϕ dependence of the form $\psi(\phi) = Ae^{im\phi}$, where m is a quantum number and A is a constant. Which of the following values of A will properly normalize the eigenfunction?

(A) $\sqrt{2\pi}$

(B) 2π

(C) $(2\pi)^2$

(D) $\frac{1}{\sqrt{2\pi}}$

(E) $\frac{1}{2\pi}$

Physics GRE Quantum Mechanics!

28. A system is known to be in the normalized state described by the wave function

$$\psi(\theta, \varphi) = \frac{1}{\sqrt{30}}(5\, Y_4^{\,3} + Y_6^{\,3} - 2\, Y_6^{\,0}),$$

where the $Y_\ell^m(\theta, \varphi)$ are the spherical harmonics. The probability of finding the system in a state with azimuthal orbital quantum number $m = 3$ is

(A) 0

(B) $\frac{1}{15}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) $\frac{13}{15}$

49. The Hamiltonian operator in the Schrödinger equation can be formed from the classical Hamiltonian by substituting

(A) wavelength and frequency for momentum and energy
(B) a differential operator for momentum
(C) transition probability for potential energy
(D) sums over discrete eigenvalues for integrals over continuous variables
(E) Gaussian distributions of observables for exact values

50. The state of a quantum mechanical system is described by a wave function ψ. Consider two physical observables that have discrete eigenvalues: observable A with eigenvalues $\{\alpha\}$, and observable B with eigenvalues $\{\beta\}$. Under what circumstances can all wave functions be expanded in a set of basis states, each of which is a simultaneous eigenfunction of both A and B?

(A) Only if the values $\{\alpha\}$ and $\{\beta\}$ are nondegenerate
(B) Only if A and B commute
(C) Only if A commutes with the Hamiltonian of the system
(D) Only if B commutes with the Hamiltonian of the system
(E) Under all circumstances

77. Consider a heavy nucleus with spin $\frac{1}{2}$. The magnitude of the ratio of the intrinsic magnetic moment of this nucleus to that of an electron is

(A) zero, because the nucleus has no intrinsic magnetic moment
(B) greater than 1, because the nucleus contains many protons
(C) greater than 1, because the nucleus is so much larger in diameter than the electron
(D) less than 1, because of the strong interactions among the nucleons in a nucleus
(E) less than 1, because the nucleus has a mass much larger than that of the electron

96. A particle of mass M is in an infinitely deep square well potential V where

$$V = 0 \quad \text{for } -a \leq x \leq a, \text{ and}$$
$$V = \infty \quad \text{for } x < -a,\ a < x.$$

A very small perturbing potential V' is superimposed on V such that

$$V' = \epsilon\left(\frac{a}{2} - |x|\right) \quad \text{for } \frac{-a}{2} \leq x \leq \frac{a}{2}, \text{ and}$$
$$V' = 0 \quad \text{for } x < \frac{-a}{2},\ \frac{a}{2} < x.$$

If $\psi_0, \psi_1, \psi_2, \psi_3, \ldots$ are the energy eigenfunctions for a particle in the infinitely deep square well potential, with ψ_0 being the ground state, which of the following statements is correct about the eigenfunction ψ_0' of a particle in the perturbed potential $V + V'$?

(A) $\psi_0' = a_{00}\psi_0,\ a_{00} \neq 0$

(B) $\psi_0' = \sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} = 0$ for all odd values of n

(C) $\psi_0' = \sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} = 0$ for all even values of n

(D) $\psi_0' = \sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} \neq 0$ for all values of n

(E) None of the above